



VARIATIONS IN THE DYNAMIC PROPERTIES OF STRUCTURES: THE WIGNER-VILLE DISTRIBUTION

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ABSTRACT

The Wigner-Ville Distribution (WVD) is a promising method for analyzing frequency variations in seismic signals, including those of interest for structural monitoring. Nonlinearities in the force displacement relationship will temporarily decrease the apparent natural frequencies of structures during strong to moderate excitation, and earthquake damage can permanently change building stiffnesses. A Fourier Transform of a building record contains information regarding frequency content, but it can not resolve the exact onset of changes in natural frequency – all temporal resolution is contained in the phase of the transform. The spectrogram is better able to resolve temporal evolution of frequency content, but has a trade-off in time resolution versus frequency resolution in accordance with the uncertainty principle. Time-frequency transformations such as the WVD allow for instantaneous frequency estimation at each data point, for a typical temporal resolution of fractions of a second. We develop a mathematical foundation for analyzing the evolution of frequency content in a signal, and apply these techniques to synthetic records from linear and nonlinear FEM analysis (including plastic rotation and weld fractures). Our analysis techniques are then applied to earthquake records from damaged buildings.

Introduction

Dynamic properties of systems can be affected by varying levels of excitation, nonlinearity in the system, and changes to the mass or stiffness of the system. Of particular interest to structural health monitoring is the identification of changes in physical properties of a system, such as a loss of stiffness caused by damage to the structural elements of a building. Investigation into finite element models and instrumented civil engineering structures has led to the application of sophisticated system identification techniques to problems in civil engineering. This paper presents a mathematical background for new signal processing techniques that the authors have successfully applied to problems in structural analysis and damage detection, with illustrative examples and applications to both numerical models and instrumented buildings.

Time-Frequency Representations of Nonstationary Signals

The Fourier Transform (FT) of a signal decomposes the original signal into harmonic components, identifying the spectral content of the signal – this process allows for system identification in terms of the natural frequencies and corresponding mode shapes, which are directly related to

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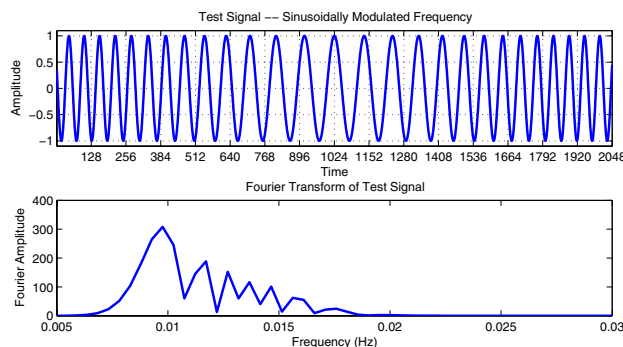
the physical properties of the system. However, when analyzing signals with evolving frequency content, the FT does not give the full information regarding the behavior of the system. A sample signal with evolving frequency is shown in Figure 1(a), along with the amplitude of the FT of the signal. While the FT correctly identifies the main components of the signal, it does not allow a straightforward identification of the onset of each signal component, or even an identification of which frequency component arrived first. All such temporal information is contained in the phase of the transform, but cannot be easily extracted for system identification purposes.

If these changes in natural frequency were related to changes in stiffness of the structure, from some damage pattern or known excitation, it would be useful to identify the extent and severity of damage, and correlate it with the temporal evolution of the behavior of the structure – perhaps associating damage or nonlinearity with measures of the input motions such as amplitude of peak ground velocity. In Figure 2, we introduce a Time-Frequency Representation (TFR) of the synthetic signal. This representation allows for a more thorough understanding of the behavior and properties of the system.

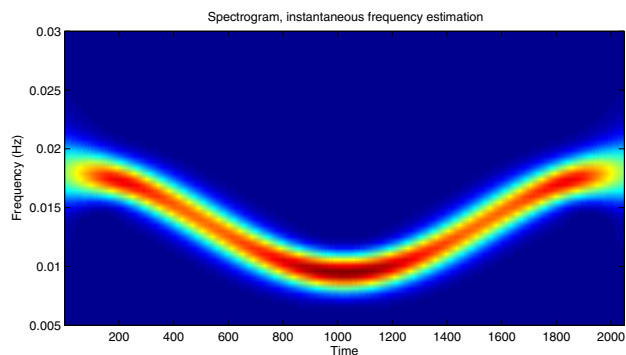
To create the time-frequency representations in Figure 2, a sliding window is used to split the signal into segments, and the Fourier Transform of each segment is then assembled into the final time-frequency matrix. Two windowing widths are presented, to emphasize the trade-off in temporal resolution versus frequency resolution. In the top figure, a wide window is used – it is immediately obvious that the temporal resolution of the representation is very coarse. In the second figure, a narrower window is used, which increases the temporal resolution, and it is now easier to identify the evolution of the frequency content contained in the signal; however, the narrow window decreases the maximum frequency resolution for each slice. Both of these windowing choices smear information in the time-frequency plane, along the time and frequency axes. The third plot in Figure 2 is an instantaneous frequency representation, an improvement over the methods in the first two plots. This method still leaks energy along both axes, but closely matches the theoretical frequency content, shown in the fourth plot. This representation, also presented in Figure 1(b), still has a noticeable block-like structure, a result of the inability of this method to create a true instantaneous energy estimation.

In time-frequency representations, it becomes straightforward to identify the different frequency components. The goal is to create a distribution which correctly identifies energy in the time-frequency plane. For the spectrogram example, the information in the time-frequency plane closely matches that of the theoretical components of the signal, shown in the last plot of Figure 2. However, the spectrogram method introduces some further complications for analyzing the temporal evolution of the frequency content of the signal. A long time window will smear the time-frequency information across the time axis, changing the perceived duration and onset of a signal component. A shorter time window, while improving temporal resolution, decreases the maximum resolution along the frequency axis. Temporal resolution and frequency resolution are inversely proportional, in accordance with the uncertainty principle, which limits the effective resolution of all Time Frequency Representations.

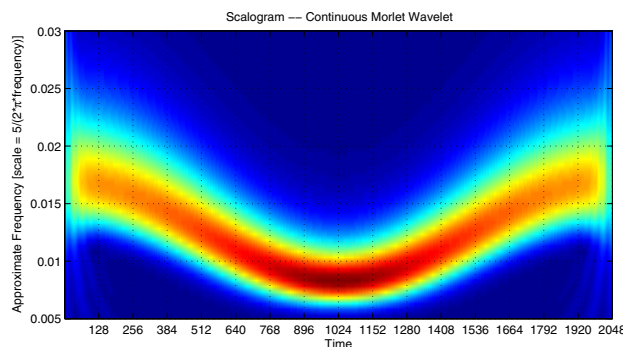
Another method for analyzing the instantaneous spectrum of a signal is to decompose it into different bases, one such choice is the wavelet transformation. The Continuous Wavelet Transform (CWT) creates a scalogram – a time-*scale* representation of the signal, where the scale of the wavelet bases has an intuitive association with frequency. The wavelet method has some advantages for time-frequency analysis. Figure 1(c) demonstrates the use of the wavelet transform on our test signal, though the frequency axis is only an approximation based on the scale of the wavelet



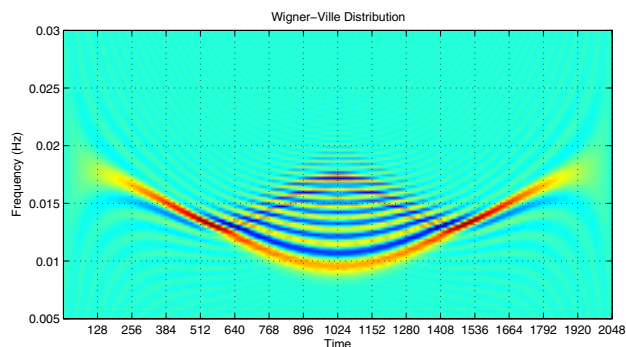
(a) Sample signal with evolving frequency, and Amplitude of the Fourier Transform – Note that the Fourier Transform, while correctly identifying the components of the signal, does not allow for a straightforward interpretation of frequency evolution.



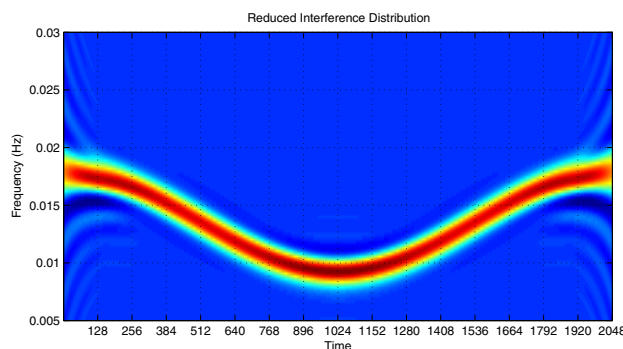
(b) Spectrogram, this distribution has limited resolution and takes on a 'blocky' structure due to the trade-off between temporal resolution and frequency resolution in the time-frequency plane – however, this representation clearly captures information about progression of the mean frequency during the signal.



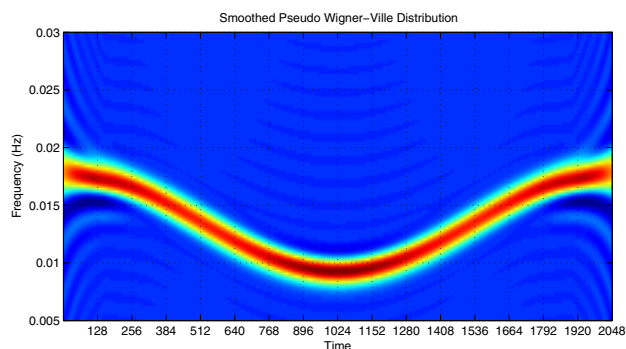
(c) Scalogram of sample signal – Continuous Morlet wavelet transformation. Wavelet transformations create a time-scale representation, where scale has an approximate equivalency to frequency.



(d) Wigner-Ville Distribution for the sample signal. Note that resolution of the signal is quite crisp, though the strong interference terms make this method unsuitable for general signal analysis.



(e) Reduced Interference Distribution for the sample signal.



(f) Smoothed Pseudo Wigner-Ville Distribution for the sample signal

Figure 1: (a) Sample signal , (b) Spectrogram, (c) Scalogram, (d) Wigner-Ville Distribution, (e) Reduced Interference Distribution, and (f) Smoothed Pseudo Wigner-Ville Distribution.

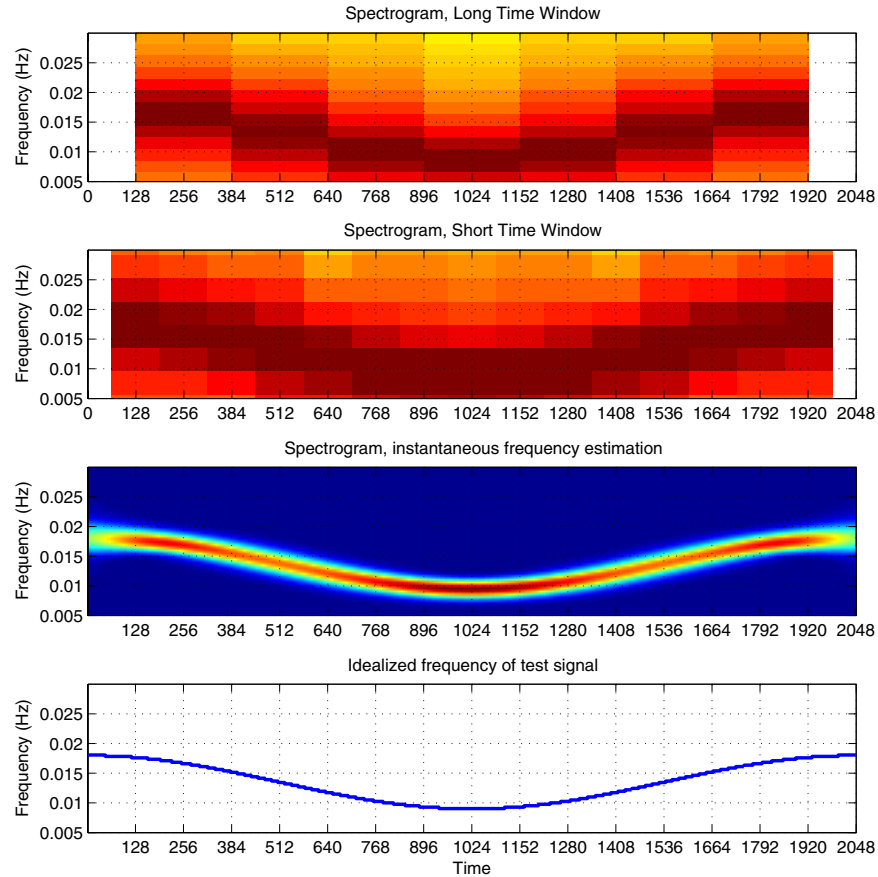


Figure 2: Time-frequency representations of sample signal: Spectrogram (long time window), Spectrogram (short time window), Spectrogram (instantaneous frequency estimation, using a Hanning windowing technique), Theoretical frequency content of the signal. These plots demonstrate the typical trade-off in time resolution against frequency resolution – A longer Fourier Transform window improves the frequency resolution, but will also smear the signal along the time axis; a shorter window will improve temporal resolution, at the cost of frequency resolution.

transform. Again, the time-frequency information roughly matches that of our known energy distribution.

Modern Time Frequency Representations

Recent signal processing advances have led to a new class of TFR tools. These new methods allow for distributions which function more accurately than the spectrogram method, though the uncertainty principle does preclude a *perfect* instantaneous frequency estimation at each point in the time-frequency plane.

The most fundamental of these TFR methods is the Wigner-Ville Distribution (WVD). The WVD is a quadratic TFR, and most other TFR methods (including the spectrogram and the scalogram) can be derived from the WVD, with a suitable choice of smoothing factors. For a

signal, $s(t)$, with analytic associate $x(t)$, the Wigner-Ville Distribution, $WVD_x(t, \omega)$ is defined as:

$$WVD_x(t, \omega) = \int_{-\infty}^{+\infty} x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) e^{-i\omega\tau} d\tau \quad (1)$$

This distribution was first introduced by E. Wigner in the context of quantum mechanics (Wigner, 1932), and later independently developed by J. Ville who applied the same transformation to signal processing and spectral analysis (Ville, 1948).

Note that the WVD is similar to the Fourier Transform, though, instead of transforming the original signal, the kernel of the WVD contains a type of autocorrelation term (in this case, the phase lag of the ambiguity function, or “...properly symmetrized covariance function...” (Flandrin and Martin, 1997)).

The analytic associate $x(t)$ of a signal $s(t)$ is here defined such that $x(t) \equiv s(t) + iH[s(t)]$, where $H[s(t)]$ is the Hilbert Transform of the signal $s(t)$. In this paper, the TFR techniques are applied to the analytic associates of real signals unless noted otherwise – in particular, $x(t)$ is generally the complex-valued analytic associate of some real-valued time signal of interest.

The Wigner-Ville Distribution for the test signal is presented in Figure 1(d). This representation has some significant differences from the spectrogram or scalogram methods. Immediately this representation gives a very clear representation of the signal, matching the theoretical energy distribution nearly exactly. This representation, however, introduces many complicated interference terms in the time-frequency plane. These interference terms are generated by the quadratic nature of the Wigner-Ville distribution; cross terms generate highly oscillatory interference in the time-frequency plane.

In addition to being an entirely real-valued function, the WVD also satisfies the marginal and total energy conditions:

$$\int_{-\infty}^{+\infty} WVD_x(t, \omega) d\omega = |x(t)|^2 \quad \text{Frequency Marginal Condition} \quad (2)$$

$$\int_{-\infty}^{+\infty} WVD_x(t, \omega) dt = |\mathbb{X}(t)|^2 \quad \text{Time Marginal Condition} \quad (3)$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} WVD_x(t, \omega) dt d\omega = \mathbb{E}_x \quad \text{Total Energy Condition} \quad (4)$$

...where $\mathbb{X}(t)$ is the Fourier Transform of $x(t)$, and $\mathbb{E}_x$ is the total energy of signal $x(t)$. The integral along the time axis is equal to the power of the signal, while the integral along the frequency axis gives the power spectrum of the signal (the squared Fourier Transform) – the energy condition states that the total area of the WVD, the double integral across time and frequency, is the energy contained in the original signal. These conditions have an intuitive appeal, as they imply a limitation on the extent of the signal in the time-frequency plane. In an ideal representation, a signal of short duration would have a narrow representation along the time axis, and the frequency content would be localized to the frequency of the signal. Meeting the marginal conditions is one way in which optimal TFR techniques can be constructed, though these conditions are neither necessary nor sufficient for the construction of useful representations.

Interference terms, however, are a significant obstacle to using the WVD as a system identification tool. Cross-term interference (from the quadratic kernel) generates highly oscillatory interference, which rapidly varies from high amplitude to low amplitude, and this interference is

often of larger magnitude than the representation of the auto-terms. Negative values, under this interpretation, would correspond to negative energy – this negates a physical interpretation of the WVD as an instantaneous energy. However, the satisfaction of the marginal conditions and many other desirable properties has led to an interest in improving the accuracy of the WVD as a TFR, primarily by reducing the effects of the interference terms on the final time-frequency result.

Refining the WVD has led to a family of Reduced Interference Distributions; In general, a Reduced Interference Distribution (RID) refers to any distribution that reduces the expression of the cross-terms relative to the auto-terms in a quadratic TFR representation (Boashash, 2003). One such RID uses a kernel based on a Hanning window and can also be reformulated to use different choices of smoothing kernel (eg. binomial, bessell, and bartlett/triangular windows). Figure 1(e) shows the RID for the test signal – while there are some artifacts from the smoothing process, this representation gives the clearest picture of the true instantaneous energy distribution of the signal. In Figure 1(f) another related distribution, the smoothed pseudo Wigner-Ville transformation, is presented to show an alternate method of reducing the cross-term interference (Bradford, 2005).

Application to Building Records

For the signals of interest to Structural Health Monitoring (SHM), the evolution of natural frequencies becomes important for damage detection and system identification. Small to moderate earthquakes, for example, will temporarily decrease the apparent natural frequencies of structures – nonlinearity in the force-displacement relationship will cause an apparent loss of stiffness with greater excitation levels. This effect can be seen in some buildings under different loading conditions, such as strong winds or forced vibration testing (eg. Caltech's Millikan Library, in Clinton (2004), Bradford (2005), and others). Buildings can also be damaged during earthquake loading, leading to permanent changes in the dynamic characteristics.

With a goal of identifying the onset of these changes in dynamic properties, the authors have developed a framework in which to apply modern time-frequency analysis techniques to data from civil structures under earthquake loading. The goal in these studies is to obtain a detailed, instant-for-instant representation of the dynamic properties of a structure, and use these changes in properties to infer damage patterns.

To validate these methods, the authors have applied Time-Frequency Representation techniques to a finite-element model that includes varying sources of nonlinearity. The finite-element program used in this study was developed by Professor J. Hall, Caltech. It is based on a planar-frame fiber model, and includes material nonlinearity, geometric nonlinearity (P- Δ effects, member buckling), and weld fractures (Hall, 1997). A 20-story steel moment-frame building, with height of 78.26m above ground, was designed according to UBC94, and input to the finite-element program. Three versions of the model: nearly elastic, nonlinear inelasticity without weld fracture, and nonlinearity including weld fracture, were subjected to the strong ground motions recorded at station HKD095 during the 2003 Tokachi-Oki earthquake (Mw 8.3). These ground motions are of sufficient amplitude to damage the building, and the resulting records from the model were then analyzed using the TFR methods described in this paper. A description of the damage state is generated during the model analysis, and comparing this data to the time-frequency information provides a validation for our efforts to correlate damage with changes in dynamic properties of the structure.

Figure 3 gives an example of the relation between information in the time-frequency plane, and the evolution of damage in the structure. Figure 4 is the TFR for the building in the linear elastic case, without plastic hinges or weld fracture. Figures 3 & 4 each also show an enlargement of the segment of record where strong shaking began.

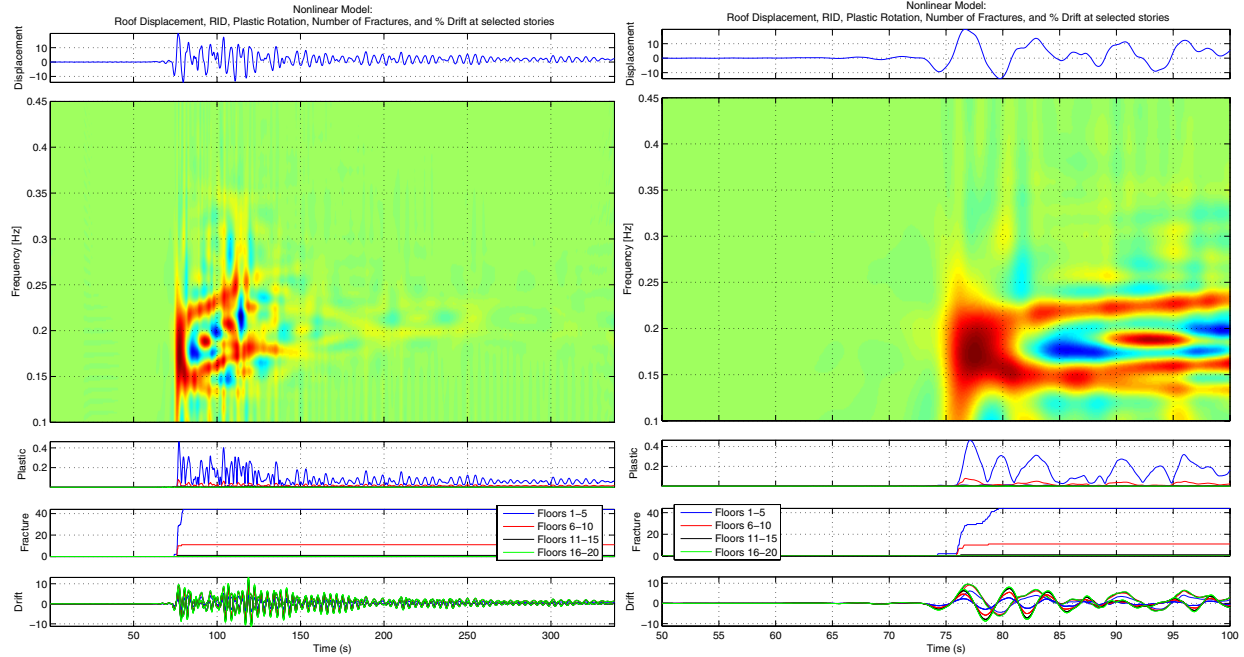


Figure 3: RID of synthetic data for the nonlinear FEM model. The top plot is the roof displacement, the large center plot is the RID of the data, and the three bottom plots present the plastic rotation, number of fractured welds, and drift for selected stories – these measures are presented for selected groups of floors, as noted in the legend. The initial strong motion pulse is magnified in the right-hand plot, to more clearly show the evolution of frequencies on a time scale of seconds, and show the correlation with the selected damage measures.

The resonant frequency of the building, under small excitations, is near .3Hz – during the event under linear conditions, it drops to around .25Hz during the strongest shaking. Under nonlinear conditions, the frequency has a more significant decrease and reaches .15Hz, a drop of $\sim 50\%$ in the original natural frequency, corresponding to a decrease of almost 75% of the global stiffness of the structure. In the nonlinear case, the change in stiffness is matched by an increase in damage measures – plastic hinges and fractured welds. Using the TFR methods presented in the paper, it is possible to examine the instant-for-instant correlation between damage and dynamic characteristics of a structure (Bradford, 2005).

In addition to synthetic building records, records from historical earthquakes provide interesting data from structures undergoing nonlinear behavior. One such building is the Imperial County Services Building in El Centro. This six-story reinforced concrete building was famously damaged during the 1979 Imperial Valley earthquake, and later demolished. At the time of the event, the building was instrumented with 13 accelerometers on 4 levels in the building, and 3 channels at a free-field site (CSMIP: <http://www.consrv.ca.gov/cgs/smip/>). In Fig-

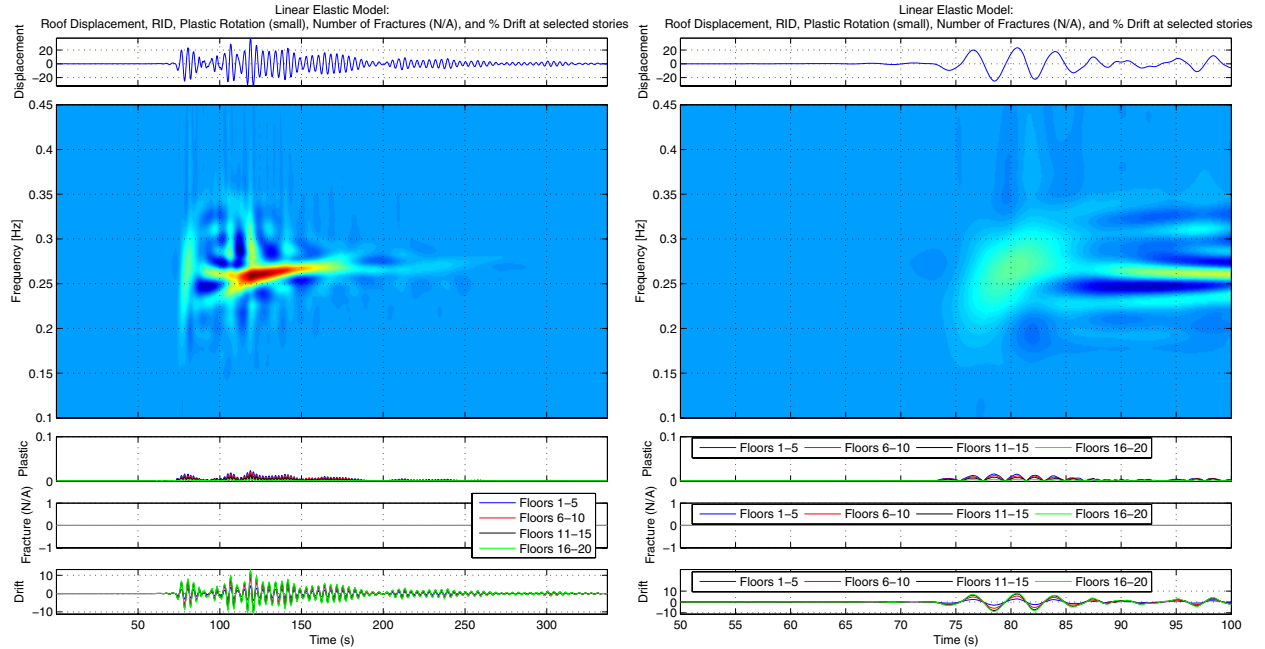
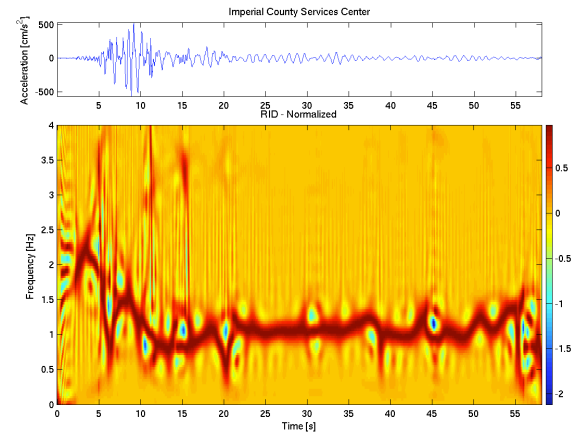
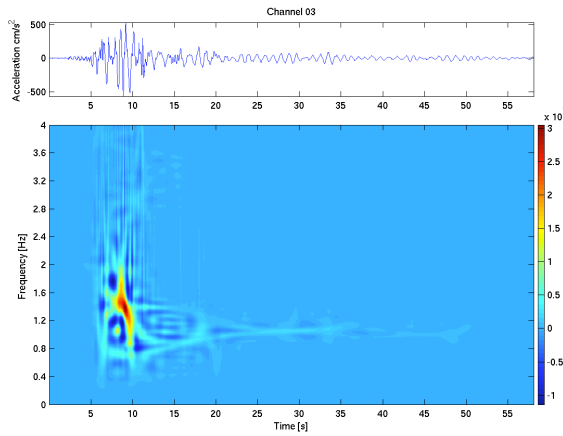


Figure 4: RID of synthetic data for the linear FEM model. (Model is not truly linear, as there is some plastic behavior of the joints. This model was created without weld fracture, and with material properties that match linear behavior as well as possible.) Plots as in Figure 3. The initial strong motion pulse is magnified in the right-hand plot, to more clearly show the evolution of frequencies on a time scale of seconds, though the change in frequency is much less noticeable than in the full nonlinear case.

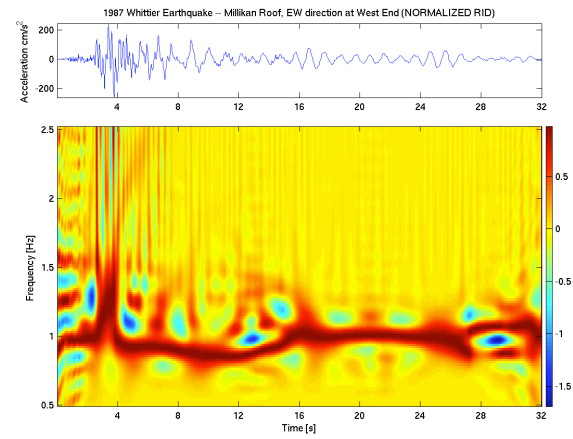
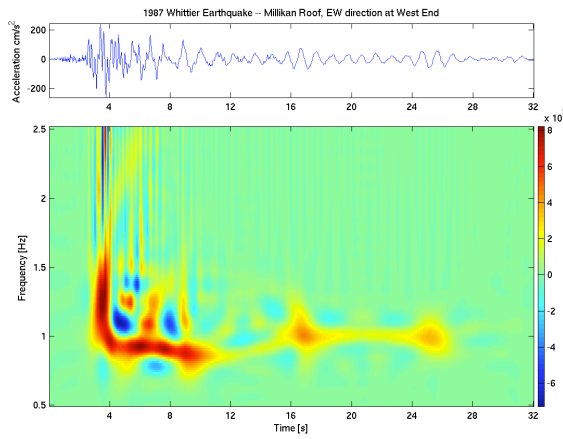
ures 5(a) & 5(b), the building suffers a significant loss of stiffness between 6 and 12 seconds into the event. Past investigations into this building have suggested similar conclusions regarding the onset of damage and the loss of the ground floor, in particular that damage was initiated ~ 7 seconds into the event, and columns collapsed ~ 11 seconds into this record (Rojahn and Mork, 1982).

Another example building is the Millikan Library, on the Caltech campus – one of the worlds most heavily researched and instrumented buildings. This nine-story reinforced concrete building has undergone strong shaking from several earthquakes, including the 1987 Whittier Narrows earthquake. This event permanently changed the natural frequencies of the Millikan Library by $\sim 2\%$ (Clinton, 2004), an RID of the event is presented in Figure 5(c), and the RID, scaled by peak amplitude at each time instant, is presented in Figure 5(d).



(a) RID of Imperial County Services Building, roof records

(b) RID of Imperial County Services Building, scaled by peak amplitude at each time instant.



(c) RID of Millikan Library 9th floor records for 1987 Whittier Narrows event

(d) RID of Millikan Library, scaled by peak amplitude at each time instant

Figure 5: Behavior of instrumented buildings during strong earthquakes. – Imperial County Services Building, El Centro, NW Component of roof response during 1979 Imperial Valley Event: a) RID b) Normalized RID – Millikan Library, EW Component of 9th Floor Response during 1987 Whittier Narrows event: c) RID d) Normalized RID – Adapted from Bradford, 2005.

Conclusions

The Wigner-Ville Distribution, and associated distributions such as the Reduced Interference Distribution, allow for a complete investigation into the evolution of frequency content in a signal, particularly the signals of interest to structural monitoring and evaluation. A variety of time-frequency representation techniques have been proposed, which provide useful information about the behavior of structures, including an estimation of damage during strong earthquake motions. The results from numerical modeling and damaged buildings verify the use of these methods for damage detection.

Acknowledgements

The authors thank the CSMIP program for the Imperial County Services Center records, Matt Muto (Caltech) for assistance with obtaining the Millikan Library / Whittier Narrows records, and Caltech Professor John Hall who developed the nonlinear FEM code used in our numerical simulations.

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